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A single-Item uncapacitated lot-sizing problem with remanufacturing and outsourcing

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Abstract

This paper addresses the single-item, dynamic lot-sizing problem for systems with remanufacturing and outsourcing. Therein, demand and return amounts are deterministic over a finite planning horizon. Demand may be satisfied by the manufacturing of new items, remanufactured items, or outsourcing, but it cannot be backlogged. The objective of this study is to determine the lot sizes for manufacturing, remanufacturing, and outsourcing that minimise the total cost, which consists of the holding costs for returns and manufactured/remanufactured products, setup costs, and outsourcing costs. The problems addressed in this paper are an extension of those addressed by Richter et al. (2000,and 2001), Teunter et al. (2006), and Aksen et al. (2003). In this paper, the separate setup costs scheme is considered, we propose a dynamic programming approach to derive the optimal solution in the case of large quantities of returned product. The complexity of this dynamic programming approach is $O(T^2)$, wherein T is the number of periods in the planning horizon.

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Keywords: Lot sizing; remanufacturing; outsource; dynamic programming

1.Introduction

Economic incentives, legal pressure, and societal pressure have motivated an increasing number of companies to engage in the product recovery business, which refers to activities that aim to regain materials and added value in used or returned products (Thorn and Rogerson 2002)[1]. A key component of product recovery is remanufacturing, which can be defined as the recovery of returned or used products and often involves disassembly, cleaning, testing, part replacement/repair and reassembly operations. The remanufactured items are as-good-as new items. Remanufacturing is of great social concern in many industries, such as single use cameras, machine tools, automobile engines, and computers. At the same

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time, remanufacturing can be profitable. For example, Volvo has established operations for the salvaging and dismantling of cars, which have led to the generation of revenue. In addition, Canon and Xerox remanufacture products that are worn out or obsolete, which is now more profitable than manufacturing new products (Stock et al. 2000)[2]. The Yuchai group is the first and largest conglomerate to remanufacture automobile engines in China. Remanufacturing development in the Yuchai group has not only secured significant economic benefits but has also resulted in good social and ecological benefits via the generation of 500 million Yuan in economic returns in 2006, reduced industrial pollutant emissions, and saved resources.

Our study is an extension of the problems that were addressed by Richter et al. (2000 and 2001) [3][4]and Teunter et al. (2006)[5], which include single-item lot sizing with manufacturing but without outsourcing. Our model differs from those that investigate lot sizing with outsourcing, because our model considers remanufacturing. This paper addresses the single-item, dynamic lot-sizing problem for systems with remanufacturing and outsourcing. Demand and return amounts are deterministic over a finite planning horizon. Demand may be satisfied by the manufacture of new items, remanufactured items, or outsourcing but cannot be backlogged. The objective of this study is to determine the lot sizes for manufacturing, remanufacturing, and outsourcing that minimise the total costs, which consist of the holding costs for returns and manufactured/remanufactured products, setup costs, and outsourcing costs. For the lot-sizing problem with remanufacturing, Van den Heuvel[6] has demonstrated that the problem becomes NP-hard when variable (re)manufacturing costs are included, even under the condition that the variable cost for manufacturing is larger than that for remanufacturing (which will typically hold if remanufacturing is economically motivated). Therefore, problems that include remanufacturing and outsourcing will be more complex, and only some specific problems can be solved in polynomial time. In this paper, t separate setup costs scheme is considered, we propose a dynamic programming approach to derive the optimal solution in the case of large quantities of returned product. The complexity of the proposed approach is $O(T^2)$, wherein T is the number of periods in the planning horizon.

This paper contributes by: (1) developing an optimisation model in order to simultaneously address several critical issues in production planning, including single-item, multi-period, remanufacturing, and outsourcing and (2) establishing the characteristics of single-item lot sizing with remanufacturing and outsourcing and developing a polynomial algorithm for the model. The rest of this paper is organised as follows. In section 2, we develop an uncapacitated, production-planning model that includes remanufacturing and outsourcing for a multi-period, single-item problem with separate setup costs. In this case, there are dedicated production lines for manufacturing and remanufacturing. In section 3, we propose a dynamic programming approach to derive the optimal solution when there are large quantities of returned products. In section 4, we provide a short conclusion and suggest future research.

2. The Mathematical Model

Assume that the number of planning periods that is under consideration is T. The length of each period may be a week or a month, depending on the application. Let T = {1, 2 …, T} be the index set of periods. The demand D_t of a product in each period $t(t \in \mathrm{T})$ is satisfied by new products, remanufactured products, or outsourcing. Products that are manufactured and remanufactured are both regarded as serviceable products, which are undistinguishable in terms of functionalities and quality. The proposed model is referred to as the single-item, uncapacitated lot-sizing problem with remanufacturing and outsourcing (SULPRO) model. The given conditions include the product demand in every period and the unit costs for manufacturing, remanufacturing, outsourcing, and setup. The SULPRO model seeks to determine the quantities for manufacturing, remanufacturing, and outsourcing and the stock level of each product in each period while meeting all demands at a minimum total cost.

The following are the definitions of the sets, parameters, and variables of the SULPRO model:

Sets:

$T = \{1, 2 \ldots T\}$: set of time periods.

Parameters:

D_t: demand in period t $\forall t \in T$.

s_t^r: setup costs of remanufacturing the returned products t $\forall t \in T$.

s_t^m :the setup costs of manufacturing new products t $\forall t \in T$.

c_t^o: unit outsourcing cost in period t $\forall t \in T$.

c_t^m: variable manufacturing cost for a product in period t $\forall t \in T$.

c_t^r: variable remanufacturing cost of a returned product in period t $\forall t \in T$.

h_t^s: unit holding cost for a serviceable product in period t $\forall t \in T$.

h_t^r: unit holding cost for a returned product in period t $\forall t \in T$.

R_t: quantity of a product returned in period t $\forall t \in T$.

Variables:

y_t: =1, if production is manufactured or remanufactured in period t; 0, otherwise $\forall t \in T$.

X_t^m: the quantity of a product manufactured in period t $\forall t \in T$.

X_t^r: the quantity of a product remanufactured in period t $\forall t \in T$.

X_t^o: the quantity of a product outsourced in period t $\forall t \in T$.

I_t^s: the inventory of a serviceable product at the end of period t $\forall t \in T$.

I_t^r: the inventory of a returned product at the end of period t $\forall t \in T$.

The problem in this section can be characterised as follows: (1) only single-item proudction is considered in this model. The demand and return amounts of the items that are remanufactured are known for all the periods of the planning horizon. (2) For one item, there are separated setup costs for manufacturing and remanufacturing. This feature is suitable if manufacturing and remanufacturing operations are performed on different production lines (see Teunter et al. 2006)[5]. The parameters s_t^r and s_t^m denote the setup costs of remanufacturing the returned products and the setup costs of manufacturing new products. (3) The holding cost-rate for the serviceable items is at least the cost of holding the returns. Because remanufacturing will add value to a product, it is a practical assumption to include the holding cost of a returned item (Teunter et al. 2006)[5]. For the same reason, in this paper, the authors further assume that the cost of manufacturing a new product is at least the cost of remanufacturing the product returned from a customer. The costs of manufacturing and remanufacturing vary as a function of period. (4) For the real marketplace, the outsourcing cost is a market price. Suppose that the gross marginal profit is nonnegative. The authors assume the cost of outsourcing is at least the cost of manufacturing new products. (5) There is a large quantity of returned products in the SULPRO model. This assumption is equal to constraint (1), as listed below. This condition means that in each period, the quantity of the available returned product is large enough to meet the demands. The details are explained in Li et al. (2006)[7].

$$R_t \geq \sum_{j=1}^{t'-1} D_j - \sum_{j=1}^{t-1} R_j, \forall t \in T \qquad (1)$$

Where t' is the first period after period t that satisfies $s^r_{t'} \le D_{t'}\{(c^r_t - c^r_{t'}) + \sum_{j=t}^{t'-1}(h^s_j - h^r_j)\}$. The following are the functions of the SULPRO model:

$$\text{SULPRO: } \min \sum_{t=1}^{T}(s^r_t y^r_t + s^m_t y^m_t + c^m_t X^m_t + c^r_t X^r_t + c^o_t X^o_t + h^s_t I^s_t + h^r_t I^r_t) \tag{2}$$

$$\text{s.t. } I^r_t = R_t + I^r_{t-1} - X^r_t, \forall t \in T \tag{3}$$

$$I^s_t = I^s_{t-1} + X^r_t + X^m_t - (D_t - X^o_t), \forall t \in T \tag{4}$$

$$X^o_t \le D_t, \forall t \in T \tag{5}$$

$$y^r_t = \begin{cases} 1, \text{if } X^r_t > 0 \\ 0, \text{otherwise} \end{cases}, \forall t \in T \tag{6}$$

$$y^m_t = \begin{cases} 1, \text{if } X^m_t > 0 \\ 0, \text{otherwise} \end{cases}, \forall t \in T \tag{7}$$

$$I^r_0 = 0, I^s_0 = 0 \tag{8}$$

$$X^r_t, X^s_t, X^o_t, I^r_t, I^s_t \ge 0, \forall t \in T \tag{9}$$

$$I^r_t + I^s_t = I_t + \sum_{j=1}^{t}(X^o_j + X^m_j) \text{ with } I_t = \sum_{j=1}^{t}(R_j - D_j), \forall t \in T \tag{10}$$

Because $I^r_t + I^s_t = I_t + \sum_{j=1}^{t}(X^o_j + X^m_j)$, the objective function of the model of SULPRO can be expressed as:

$$\sum_{t=1}^{T}\left\{s^r_t y^r_t + s^m_t y^m_t + c^m_t X^m_t + c^r_t X^r_t + c^o_t X^o_t + h^r_{tT}(x^m_t + x^o_t) + H'_t I^s_t\right\} + \sum_{t=1}^{T} h^r_t I_t \tag{11}$$

where $H'_t = h^s_t - h^r_t$. The total inventory cost for the returned products over the periods t, t+1…, T is ($h^r_{tT} = \sum_{j=t}^{T} h^r_j$).Because ($\sum_{t=1}^{T} h^r_t I_t$) is a constant, it can be omitted. The following is the "relaxed SULPRO model," which is denoted as RSULPRO.

$$\text{RSULPRO-1: } \sum_{t=1}^{T}\left\{s^r_t y^r_t + s^m_t y^m_t + c^m_t X^m_t + c^r_t X^r_t + c^o_t X^o_t + h^r_{tT}(x^m_t + x^o_t) + H'_t I^s_t\right\} \tag{12}$$

The above RSULPRO model is subject to constraints (3)-(9).

3. RSULPRO with Separate Set-up Costs

Theorem 1: The optimal solution of RSULPRO is also the optimal solution of SULPRO.

The RSULPRO problem has a larger feasible region of solutions than that of the original SULPRO problem. Therefore, if the authors can prove that the optimal solution for RSULPRO is a feasible solution for SULPRO, the optimal solution for RSULPRO must be an optimal solution for SULPRO. To achieve this, it will suffice to show that the I_t^r that is generated from the optimal solution for RSULPRO meets the constraints on I_t^r. For the assumption of the quantity of available returned product [i.e., equation (1)], the following (13) is true.

$$I_t^r = \sum_{j=1}^{t}(R_j - X_j^r) \ge \sum_{j=1}^{t'} D_j - \sum_{j=1}^{t} X_j^r \ge 0 \qquad (13)$$

At the same time, the variables maintain the inventory balance of returned items. This relationship is shown in the following steps of function (14).

$$I_t^r = I_{t-1}^r - X_t^r + R_t \qquad (14)$$

This completes the proof. ■

Based on the equivalent RSULPRO model, the authors can obtain some properties for the SULPRO problem.

Lemma 1**:** The basic solution of the SULPRO model is given by:

$$X_t^r . X_t^m = 0 \text{ and } I_{t-1}^s.(X_t^r + X_t^s) = 0, \forall t \in T \qquad (15)$$

Lemma 1 is named the "zero-inventory-property" lemma in Richter et al. (2000 & 2001) [3][4]and Li et al. (2006)[7]. For optimal solutions, there can never be both manufacturing and remanufacturing activities in the same period. In addition, an activity only takes place if the inventory of the serviceable products in the previous period is zero. Later in this paper, the authors will show that this property will still hold in RSULPRO. This proof process is similar to that described in Li et al. (2006)[7].

RSULPRO can be regarded as a two-step problem. In the first step, the authors determine the optimal values of ($X_t^{o*}, \forall t \in T$). In the second step, with a set of known values ($X_t^{o*}, \forall t \in T$), the following sub-problem model of RSULPRO can be determined by (16-21):

$$\min \sum_{t=1}^{T} \left\{ s_t^r y_t^r + s_t^m y_t^m + c_t^m X_t^m + c_t^r X_t^r + c_t^o X_t^{o*} + h_{tT}^r (x_t^m + x_t^o) + H_t^{'} I_t^s \right\} \qquad (16)$$

$$\text{s.t. } I_t^s = I_{t-1}^s + X_t^r + X_t^m - (D_t - X_t^{o*}), \forall t \in T \qquad (17)$$

$$y_t^r = \begin{cases} 1, \text{if } X_t^r > 0 \\ 0, \text{otherwise} \end{cases}, \forall t \in T \qquad (18)$$

$$y_t^m = \begin{cases} 1, \text{if } X_t^m > 0 \\ 0, \text{otherwise} \end{cases}, \forall t \in T \qquad (19)$$

$$I_0^r = 0, I_0^s = 0 \qquad (20)$$

$$X_t^r, X_t^m, I_t^r, I_t^s \ge 0, \forall t \in T \qquad (21)$$

It is obvious that this sub-problem is the same as the model in Equation (11) in Richter et al. (2000)[3], except that variable costs for the manufactured and remanufactured products are considered in our model. From the discussion in Richter et al. (2001)[4], the variable costs have no influence on the property. The feature of $H_t^{'} \geq 0$ is due to the assumption of the holding cost-rate of the serviceable and returned items; hence, all of the coefficients in the objective function of the model are nonnegative. The model is expressed in functions from (16) to (21). Due to lemma 3 in Richter et al. (2001)[3], the optimal solution for RSULPRO satisfies $X_t^r . X_t^m = 0$ and $I_t^s .(X_t^r + X_t^m) = 0, \forall t \in T$. Meanwhile, the optimal solution for RSULPRO also satisfies the zero-inventory-property lemma according to Theorem 1. ■

***Lemma 2*:** $X_t^{o*} X_t^{m*} = 0, \forall t \in \{1, 2, \cdots, T\}$

Lemma 2 suggests that there is an optimal solution so that demand in a given period will be fully satisfied if manufacturing is made in that period. This lemma is proposed by Aksen et al. (2003) [8]in the single-item lot-sizing problem with lost sales. In this paper, the authors will show that this lemma is still true in the SULPRO model. The proof of Lemma 2 is similar to that of Aksen et al. (2003)[8],and the process for this proof is omitted.

***Lemma 3*:** The authors distinguish the following two cases for the optimal solution of SULPRO. Let ($D_{tT} \quad {}_{i\ t}^{T} D_i$).Case (1): Assume ${}_{i\ t}^{T} h_i^r \quad c_t^m \quad c_t^r \quad 0$. If $s_t^m \quad D_t ({}_{i\ t}^{T} h_i^r \quad c_t^m \quad c_t^r) \quad s_t^r$, there is an optimal solution when $x_t^m \quad 0$. If $s_t^m \quad D_{tT} ({}_{s\ t}^{T} h_s^r \quad c_t^m \quad c_t^r) \quad s_t^r$, and there is an optimal solution when $x_t^r \quad 0$ .Case (2): Assume ${}_{i\ t}^{T} h_i^r \quad c_t^m \quad c_t^r \quad 0$. If $s_t^m \quad D_t ({}_{i\ t}^{T} h_i^r \quad c_t^m \quad c_t^r) \quad s_t^r$, there is an optimal solution when $x_t^r \quad 0$. If $s_t^m \quad D_{tT} ({}_{s\ t}^{T} h_i^r \quad c_t^m \quad c_t^r) \quad s_t^r$, and there is an optimal solution when $x_t^m \quad 0$. The process is omitted.

Let f_{it} be the minimal cost to cover the demand for the periods *i*+1, *i*+2 … *t* by either one remanufacturing or one manufacturing activity. The minimal cost can be calculated with the Wagner-Whitin algorithm. The setup cost is shown in function (22):

$$
\begin{aligned}
& f_0 \quad 0 \\
& f_t \quad \min_{0\ i\ t} \{ c_{it} \quad f_{it} \quad f_i \} \\
& c_{it} \quad {}_{j\ i\ 1}^{t\ 1} H_j^{'} D_{j\ 1,t}, i \quad 0,1,...,t \quad 1, t \quad 1,2,...,T
\end{aligned} \tag{22}
$$

Determining the cost of f_{it} is the key to solving the RSULPRO problem with a large initial quantity of low-value returned items. The following are some definitions that can be used to solve f_{it} .

Definition 8: MC_{jt}^r denotes the trade-off between the marginal costs of outsourcing in period *t* and of remanufacturing in *j* before *t*. The value of MC_{jt}^r is determined by function (23).

$$
MC_{jt}^r \quad \min \ c_t^o , c_j^r \quad h_{jt}^s \quad h_{jt}^r \tag{23}
$$

Definition 9: MC_{jt}^{m} denotes the trade-off between the marginal costs of outsourcing in period *t* and of manufacturing in *j* before *t*. The value of MC_{jt}^{m} is determined by function (24).

$$MC_{jt}^{m} \quad \min \quad c_t^o, c_j^m \quad h_{jt}^s \qquad (24)$$

Definition 10: ${}_{j}^{r}$ denotes the linked list of periods in which the demands should be met by remanufacturing. The set of periods in ${}_{j}^{r}$ is obtained by function (25).

$${}_{j}^{m} \quad \{ \quad [j \quad 1,T]: c_j^r \quad h_j^s \quad h_j^r \quad c^o\} \qquad (25)$$

From function (25), $x_t^r \qquad \underset{{}_{j}^{r}}{D}$.

Definition 11: ${}_{j}^{m}$ denotes the linked list of periods wherein the demands should be met by manufacturing. The set of periods in ${}_{j}^{m}$ is obtained by function (26).

$${}_{j}^{m} \quad \{ \quad [j \quad 1,T]: c_j^m \quad h_{jT}^s \quad c^o\} \qquad (26)$$

From function (26), $x_t^m \qquad \underset{{}_{j}^{m}}{D}$.

According to Lemma 3**,** manufacturing and remanufacturing will not simultaneously occur. In order to calculate the cost of f_{it}, only the costs of manufactured or remanufactured items with outsourcing need to be separately compared. There are two cases of *j*. For case (1), there is a remanufacturing period in period *j*. For example, when $\overset{T}{}_{i \ t} h_i^r \quad c_t^m \quad c_t^r \quad 0$ and $s_t^m \quad D_t(\overset{T}{}_{i \ t} h_i^r \quad c_t^m \quad c_t^r) \quad s_t^r$ or $\overset{T}{}_{i \ t} h_i^r \quad c_t^m \quad c_t^r \quad 0$ and $s_t^m \quad D_{tT}(\overset{T}{}_{s \ t} h_i^r \quad c_t^m \quad c_t^r) \quad s_t^r$. For case (2), there is a manufacturing period in period *j*. For example, when $\overset{T}{}_{i \ t} h_i^r \quad c_t^m \quad c_t^r \quad 0$ and $s_t^m \quad D_t(\overset{T}{}_{i \ t} h_i^r \quad c_t^m \quad c_t^r) \quad s_t^r$ or $\overset{T}{}_{i \ t} h_i^r \quad c_t^m \quad c_t^r \quad 0$ and $s_t^m \quad D_{tT}(\overset{T}{}_{s \ t} h_i^r \quad c_t^m \quad c_t^r) \quad s_t^r$.

For case (1), there is only a remanufacturing in period *j*. The values of f_{it} are obtained via function (27).

$$f_{it} \quad s_{i \ 1}^r \quad \overset{t}{\underset{j \ i \ 1}{}} D_j MC_j^r \quad h_{jT}^r (D_{it} \quad x_{i \ 1}^r) \qquad (27)$$

For case (2), there is only manufacturing in period *j*. The values of f_{it} are obtained by function (28).

$$f_{it} \quad s_{i \ 1}^m \quad \overset{t}{\underset{j \ i \ 1}{}} D_j (MC_j^m \quad h_{jT}^r) \qquad (28)$$

When the values of f_{it} are obtained via functions (27) or (28), the optimal cost of RSULPRO can be determined by function (22), and the optimal cost of SULPRO can be determined by $f_T + \sum_{t=1}^{T} h_t^r I_t$. Note that the loops in the algorithm have at most two levels, resulting in the computational complexity of O(T^2).

4. Conclusions

In conclusion, the cost of variable outsourcing was included into the framework of the lot-sizing problem with remanufacturing (Richter et al. 2000 and 2001, and Teunter et al. 2006)[3][4][5]. In the extended model, the demand and return amounts are deterministic over a finite planning horizon. The demand may be satisfied by the manufacturing of new items, remanufactured items, or outsourcing. The backlogging of demand is prohibited. The objective is to determine the lot sizes for manufacturing, remanufacturing, and outsourcing that minimise the total cost, which consists of holding costs (for returns, manufactured products, and remanufactured products), setup costs, and outsourcing costs. The authors proposed a dynamic programming approach to derive an optimal solution with large quantities of a returned product. The complexity of this algorithm is $O(T^2)$. Possible future research consists of investigating models with limited inventory capacities and multiple items.

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